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Technical annex of European Policy Brief 6 "How EU Fiscal Norms Will Become a Safety Net for the Failure of National Golden Rules"

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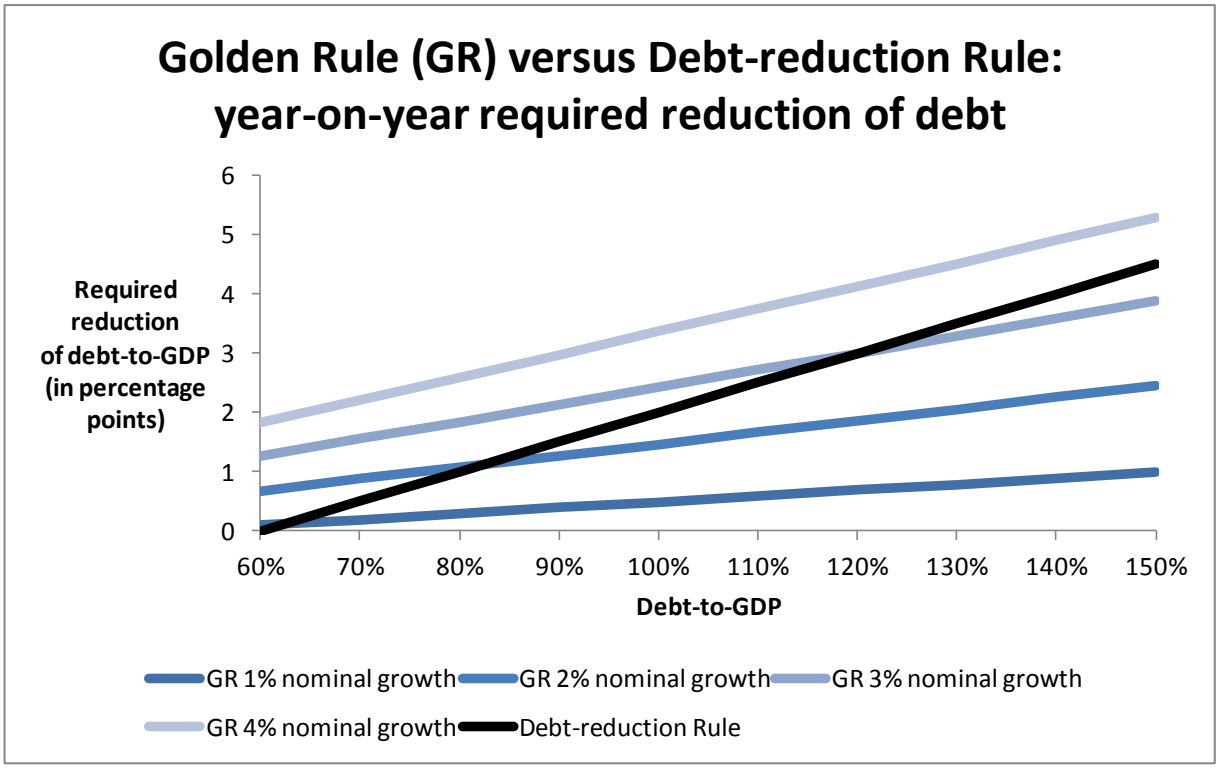
This technical annex provides background to the comparison between the Golden Rule and the Debt-reduction Rule, discussed in European Policy Brief 6. It provides both for intuitive and mathematical proof of figure 1 of that Policy Brief, which compares the stringency of the Golden Rule to that of the Debt-reduction Rule.

Intuitive Proof

The figure on page two of this technical annex illustrates the average yearly-required debt-reduction for both the Debt-reduction Rule and the Golden Rule. The black line represents the Debt-reduction Rule. The debt-reduction that is required by this Rule is independent of economic growth. This is different for the Golden Rule, where the debt-to-GDP reduction is also based on economic growth. The blue lines therefore indicate the decrease in the debt-to-GDP level that is required by the Golden Rule given different nominal growth levels (ranging from 1% to 4% nominal growth).

Given a specific nominal growth level, the Debt-reduction Rule becomes more stringent only if a country exceeds a specific debt-to-GDP threshold. This is shown in the figure on page two. For example, given a nominal growth level of 2%, the Debt-reduction Rule becomes more demanding only if a country has debt that exceeds 82.5% of GDP. If nominal growth is 3%, the tipping point is 120.5% of debt-to-GDP; in case of 4%, nominal growth the tipping point is a stunning 218.5% debt-to-GDP.



Source: own calculations

We can use these intersects to identify when the Debt-reduction Rule and the Golden Rule are equally restrictive. The curve in figure 1 of European Policy Brief 6 is composed of these intersects.

Mathematical Proof

In this mathematical proof, we provide an equation for which, given a growth and debt/GDP level, both the Debt-reduction Rule and the Golden Rule are equally strict. As the Golden Rule requires structural deficits of 0.5% of GDP or less, this implies that the Debt-reduction Rule should also impose a deficit limit of 0.5% of GDP at most.

We use the following symbols:

- B = government debt;
- GDP = Gross Domestic Product;
- g = nominal growth-to-GDP rate in year 0;
- d = government deficit in year 0;
- Subscripts indicate the relevant year. If no indication, the variable concerns year 0.

Assumptions:

$$B_0 + d = B_1$$

$$GDP_0 + GDP_0 * g = GDP_1$$

The debt-reduction rule implies that:

$$\frac{B_0}{GDP_0} - \frac{\left({}^{B_0}/_{GDP_0}\right) - 60\%}{20} = \frac{B_1}{GDP_1}$$

We can use the assumptions to modify B_1 and GDP_1:

$$\frac{B_0}{GDP_0} - \frac{\left({}^{B_0}/_{GDP_0}\right) - 60\%}{20} = \frac{B_0 + d}{GDP_0 + GDP_0 * g}$$

As we want to know in what case the Golden Rule and the Debt-reduction Rule are equally strict, we know that:

$$d = 0.5\% * GDP_0$$

Therefore:

$$\frac{B_0}{GDP_0} - \frac{\left({}^{B_0}/_{GDP_0}\right) - 60\%}{20} = \frac{B_0 + (0.5\% * GDP_0)}{GDP_0 + GDP_0 * g}$$

We can isolate g in a few steps:

$$\frac{\frac{B_0}{GDP_0} - \left[\frac{\left({}^{B_0}/_{GDP_0}\right) - 60\%}{20}\right]}{B_0 + (0.5\% * GDP_0)} = \frac{1}{GDP_0 + GDP_0 * g}$$

We now reverse the numerators and the denominators:

$$\frac{B_0 + (0.5\% * GDP_0)}{\frac{B_0}{GDP_0} - \left[\frac{\left({}^{B_0}/_{GDP_0}\right) - 60\%}{20}\right]} = GDP_0 + GDP_0 * g$$

We bring GDP_0 to the left-hand side of the equation:

$$\frac{B_0 + (0.5\% * GDP_0)}{\frac{B_0}{GDP_0} - \left[\frac{\left({}^{B_0}/_{GDP_0}\right) - 60\%}{20}\right]} - GDP_0 = GDP_0 * g$$

This results in:

$$\left.\frac{B_0 + (0.5\% * GDP_0)}{\frac{B_0}{GDP_0} - \left[\frac{\left({}^{B_0}/_{GDP_0}\right) - 60\%}{20}\right]} - GDP_0 \right/ GDP_0 = g$$